The Trailing Edge

October 2024

Gust Loading, Airspeed, and Wing Loading

(*This is an article that I intended to write long ago. I conceived it back probably before 2020, and it was still being carried as a future article for the EAA Chapter 1000 newsletter "The Leading Edge" when the chapter folded in November 2021. Part of the reason that it dragged on for so long was that I knew the answer was something I had learned in a Structures class in 1982, but since I didn't use it beyond that, I couldn't remember the details. I finally figured it out by consulting the very textbook that I used in that class. Now it has finally bubbled to the top to be written and published. Part of the reason that I write these articles is that the effort of researching the topic helps me to understand it better myself. There are at least three other such ideas left from the Chapter 1000 days that are still waiting to be written.*)

Dang! It's been another afternoon of getting beaten up in the tow plane. It's bad enough to have glider pilots yanking your tail around, but it's worse having the turbulence shake you around, bouncing off of various parts of the cockpit. Perhaps I'm not as good of a pilot as you, because I really don't enjoy getting bounced around by turbulence. I can deal with it okay in the Piper PA-25 Pawnee tow plane, but it makes me more nervous in my own Bearhawk. I think the difference is that I can sense that the Bearhawk is a little less stable overall, and I think my brain is secretly scared of hitting a bump and losing control. After all, it has happened at least once. (Ref 1)

Somewhat empirically, I figured out that the turbulence bumps seemed to be worse the faster you go. However, I could fly in the same airspace on the same day, and the turbulence bumps were pretty bad in my Bearhawk, but in a glider they were just slightly annoying. In fact, in a glider, the bumps could be indicators of rising air ("lift") which glider pilots are usually seeking out. On the other hand, back in 1990 I had a flight in an FB-111. I remember we were screaming along at some high speed, probably 400 to 500 knots or more, within 1000 feet of the ground. The ride was so smooth that it felt like sitting at my desk, except with a helmet and mask on my head.

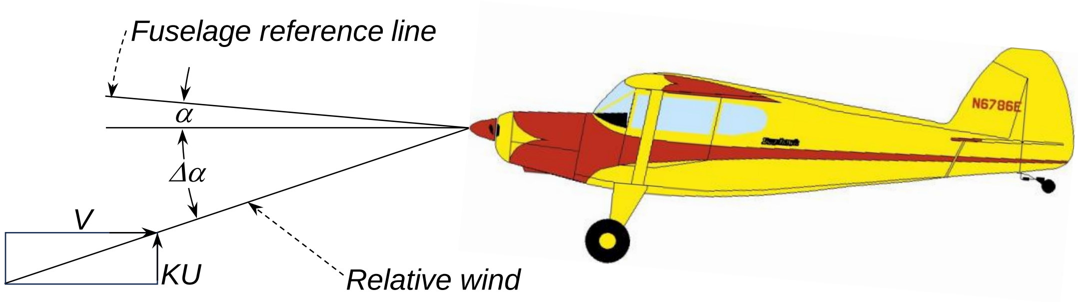
So, what was going on? The glider example would indicate that going slow was the key to reducing bumps from turbulence, but the FB-111 was so smooth at high speed. A possible clue was found in an old memory from Aero 372 in the spring of 1982. This was my structures class, and to determine the loads we needed to design for, we looked at the static requirements as shown in the V-n diagram. In addition to that, we had to design for dynamic gust loads, assuming the effects of a 50 feet/second vertical gust. I remembered that this effect was linear, getting worse with airspeed. I couldn't remember why it was linear, when most loads changed with velocity squared. This was the first time in any aeronautical engineering class that we didn't assume the atmosphere was quiescent (still and unmoving).

Back to School

To answer these quandaries, it was time to do some recreational maths. Fortunately, I still remembered what textbook was used for that class. Even better, I still had that textbook (Ref 2). I opened it up and managed to find the section that covered gust loading.

When an airplane is in level flight in calm air, the angle of attack (α) is measured from the fuselage reference line to the relative wind. Since the airplane is in level flight, the relative wind is horizontal. If the airplane suddenly strikes an ascending air current which has a vertical velocity (KU)[1], the angle of attack is increased by the angle $\Delta\alpha$, as shown in the following figure.

[1] K is the gust effectiveness factor, usually 0.8 to 1.2. U is the assumed gust velocity.



We will use the small angle assumption that α = tan α for small angles (as measured in radians). Therefore, the incremental change in angle of attack can be expressed as

$$\Delta\alpha = \frac{(KU)}{V}$$

The lift curve slope, C_{L_α}, is given by the change in lift coefficient for a change in angle of attack.

$$C_{L_\alpha} = \frac{\Delta C_L}{\Delta\alpha}$$

Rearranging for lift coefficient

$$\Delta C_L = C_{L_\alpha}\Delta\alpha$$

Substituting our expression for the change of angle of attack given by gust velocity

$$\Delta C_L = \frac{C_{L_\alpha}(KU)}{V}$$

The change in lift force resulting from a change in lift coefficient is given by

$$\Delta L = \Delta C_L \frac{\rho V^2}{2} S$$

Load factor is the lift divided by the airplane weight, so dividing this expression by weight will give us the incremental change in load factor, which is the change in "g" that we feel.

$$\Delta n = \Delta C_L \frac{\rho V^2}{2} S \frac{1}{W}$$

To see how the change in load factor is related to the gust velocity, substitute in the expression derived above

$$\Delta n = \frac{C_{L_\alpha}(KU)}{V} \frac{\rho V^2}{2} S \frac{1}{W}$$

Divide the Weight (W) by planform area (S) to represent wing loading. Finally, divide out[2] one of the airspeeds (V) in the numerator with the airspeed in the denominator to get

$$\Delta n = C_{L_\alpha}(KU) \frac{\rho V}{2} \frac{1}{\left(\frac{W}{S}\right)}$$

[2] My algebra teacher would not let us say "cancel"

This is a critical step, because the result is that the gust response (Δn) is a **linear function of airspeed** and not a function of the square of the velocity like most forces are.

For a single airplane, in this equation, lift curve slope (C_{L_α}) and two (2) are constants. Since the change in altitude is very small, density (ρ) can be considered constant. Over this short time interval, weight (W) can be considered constant. Thus, the load factor response to a gust is a function of airspeed (V), the gust speed (KU), and the wing loading (W/S).

From this equation, we can see that the gust response, i.e. how hard the kick in the butt is, for a given gust velocity will increase as airspeed increases. This is consistent with advice to slow down when encountering turbulent air. The slower you can fly, the less the perceived intensity of the turbulence. This equation also shows that aircraft with a higher wing loading will be less responsive to turbulence.

So what does that mean to me?

Let's run some numbers for the cases mentioned earlier. Per Reference 2, a reasonable design gust velocity (KU) is 30 feet/second.

Picking some reasonable flight conditions, we can calculate our Δn for these conditions. Given the weight (W) and wing planform area (S), we can calculate the wing loading with this incredibly intricate equation.

$$\left(\frac{W}{S}\right) = \frac{W}{S}$$

To find the lift curve slope, we will assume a 2-D lift curve slope (C_{l_α}) of 2π/radian, and estimate the 3-D lift curve slope using the Aspect Ratio. To calculate the Aspect Ratio, we use the wing span (b) and the planform area (S).

$$AR = \frac{b^2}{S}$$

The 3-D lift curve slope (C_{L_α}) is estimated by

$$C_{L_\alpha} = \frac{C_{l_\alpha}}{\left[1 + \left(\frac{C_{l_\alpha}}{\pi e AR}\right)\right]}$$

For this calculation, the Oswald's efficiency factor (e) was assumed as "1". This will have minimal effect on the results and is not worth the time to try to figure out the actual value.

For density (ρ) standard day was assumed and the density was calculated from the density altitude (H_d).

$$\rho = \rho_{SL}*(1\text{-}6.87559*10^{-6}\ H_d)^{4.2559}$$

Therefore, our calculations are represented here in this table.

Aircraft	True Airspeed knots	Gust Speed ft/sec	Altitude feet	Gross Weight pounds	Wing Area sq feet	Wing Span feet	Wing Loading lb/ft2	Aspect Ratio	Lift Curve Slope CL/radian	Density slug/ft3	True Airspeed ft/sec	Delta n
ASK-21	61	30	6000	1294	193	56.0	6.7	16.25	5.595	0.001987	103	2.56
Bearhawk	125	30	6500	2400	180	33.2	13.3	6.11	4.734	0.001957	211	2.20
FB-111	600	30	3500	80000	525	32.0	152.4	1.95	3.102	0.002143	1013	0.66

Several interesting conclusions can be found in this table. The first is that we see confirmation that the FB-111 was a smooth ride in turbulence. A 30 ft/sec vertical gust would only create a 0.66 g bump. That's from 1 g to 1.66 g. Compare that to what the same gust will do to the Bearhawk or the ASK-21 glider – from 1 g to over 3 g! Even though the FB-111 is going ridiculously fast by comparison, its very high wing loading makes it very unresponsive to gusts. Also helping to reduce the responsiveness to gusts is the low lift curve slope, caused by the very low aspect ratio of the essentially delta wing when fully swept. This low gust responsiveness is very convenient when penetrating at very low altitudes. Other fighter aircraft with lower wing loadings did not have anywhere near as smooth of a ride.

Strangely, the ASK-21 glider and the Bearhawk have roughly the same responsiveness to the gust according to this equation. This can be explained mathematically because while the glider is at roughly half the airspeed of the Bearhawk, the glider also has about half the wing loading of the Bearhawk. These two effects counter each other, resulting in roughly the same output.

But how can this be, when the experiential data state that the bumps in the glider are much softer than in the Bearhawk? I think the answer lies in the construction methods. The glider has a fiberglass wing, which is less stiff (more flexible) than the aluminum Bearhawk wing. The fiberglass wing can be seen to flex under gust loads. This flexing absorbs some of the gust load, transmitting less of it to the cockpit. The Schweizer SGS 2-33 has an aluminum wing similar to the Bearhawk (also a wing loading less than the ASK-21), and thus seems more responsive to gusts.

As for the size of the gust used for this calculation, it turns out that it is rather large. 30 ft/sec multiplied by 60 seconds/minute means this gust is a vertical velocity of 1800 feet/minute. Most glider pilots are excited to see 1000 feet/minute of lift, which is a really good soaring day. More likely is around 600 feet/minute of lift, which is one third the size of our test gust. Then again, remember that this is a number used for designing structures, which means it is intended to be the extreme of conditions actually seen in nature.

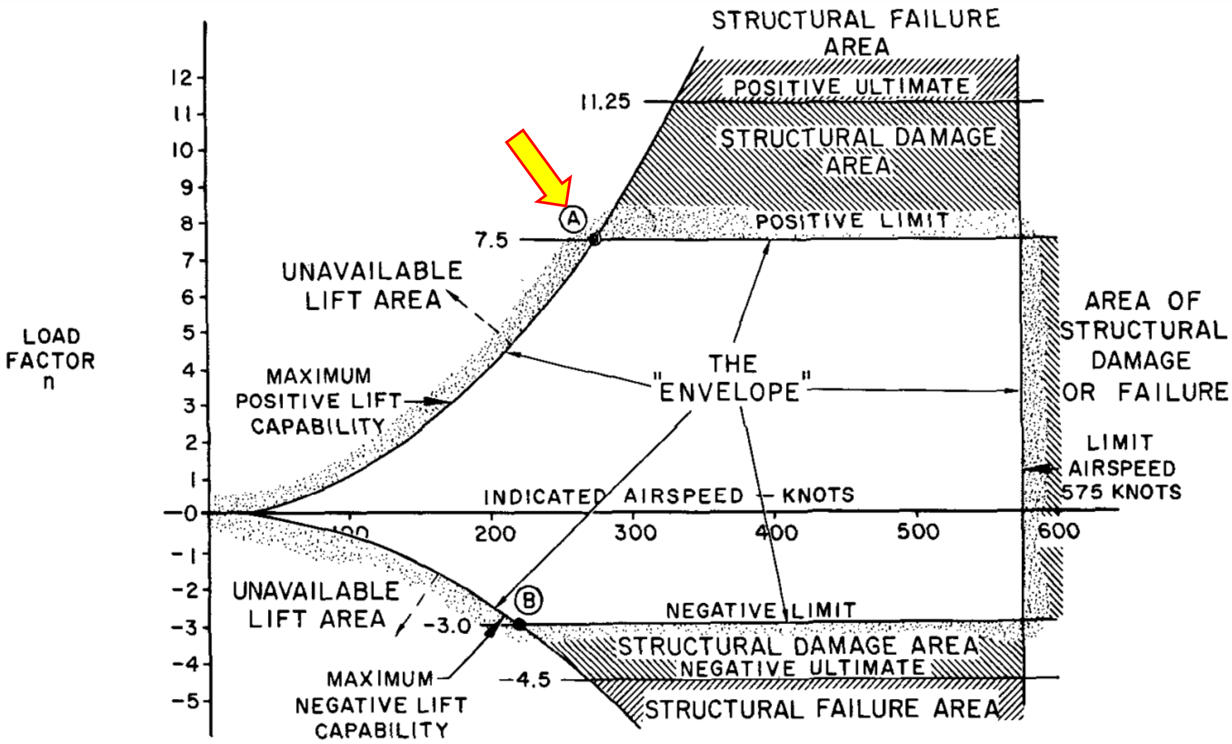
Considering that gusts can be down as well as up, for a down gust our calculations show the cockpit load factor for the Bearhawk going from 1g to -1.2g, which will certainly get the pilot's attention as well as throw things around the cockpit. Just such a gust was experienced in the Bearhawk back in 2009 when unintentionally flying a little too close to a thunderstorm.

Cleared To Maneuver

Now that we have an idea about how gusts affect our aircraft, what can we do with this knowledge to make piloting decisions?

One of the most misunderstood numbers regarding maneuvering is V_A "design maneuvering speed". A good definition of design maneuvering speed comes from Rod Machado, "The design maneuvering speed (V_A) is the speed at which the airplane will stall before exceeding its design limit-load factor in turbulent conditions or when the flight controls are suddenly and fully deflected in flight." (Ref 3) I still can't imagine why any sane pilot outside of an aerobatic routine would willingly yank the flight controls to full deflection while in flight. In any case, the key point is that the wing will stall before exceeding the limit load factor, regardless of what caused the sudden increase in angle of attack.

While maneuvering speed can apply to load limits in the lateral or directional axes, it is generally thought of in the longitudinal (pitch) axis. In this case, the maneuvering speed is the same speed as the corner speed, as shown by Point A on this V-n diagram (as originally depicted in *Aerodynamics For Naval Aviators* (Ref 4) and replicated in many other publications since).



Design maneuvering speed decreases with gross weight, as discussed in detail in http://eaa1000.org/1803nltr.pdf#page=7 (Ref 5). This change is given by the equation

$$V_A = V_{A_o}\sqrt{\frac{W}{W_{max}}}$$

V_A	Current maneuvering speed
V_{A_o}	Maneuvering speed at max gross weight (published)
W	Current weight
W_{max}	Maximum gross weight

As discussed in Reference 5, the wing bending moment and wing structure are not the only considerations when looking at maneuvering speeds. Even though the wing can produce more load factor with the same lift at light weights, it is critical not to exceed the limit load factor so that installed equipment, such as the engine, doesn't get ripped out of its mounts by excessive load factor.

Another gust loading consideration is the maximum speed that the structure can withstand a specified vertical gust. That is, the speed at which a specified vertical gust will increase the load factor to the limit load factor without stalling the aircraft. This is called V_B, the design speed for maximum gust intensity.

Based on the old 14 CFR Part 23, *Airworthiness Standards: Normal Category Airplanes*[3], §23.333 Flight envelope:

[3] The current version of 14 CFR Part 23, *Airworthiness Standards: Normal Category Airplanes*, released on 27 June 2024, has been totally re-written to provide performance based requirements. That is, it defines what the manufacturer must prove to the FAA that the airplane can do without telling the manufacturer how to do it. Previous versions of 14 CFR Part 23 were prescriptive, that is, the regulation told the manufacturer how the airplane had to be built. While the new version allows much more flexibility for incorporating new technologies, the old version is a useful reference for what should be expected. Thus, all "requirements" quoted in this article will be from the "old" Part 23.

(c) *Gust envelope*. (1) The airplane is assumed to be subjected to symmetrical vertical gusts in level flight. The resulting limit load factors must correspond to the conditions determined as follows:

(i) Positive (up) and negative (down) gusts of 50 f.p.s. at V_C must be considered at altitudes between sea level and 20,000 feet….

(ii) Positive and negative gusts of 25 f.p.s. at V_D must be considered at altitudes between sea level and 20,000 feet….

(2) The following assumptions must be made:

(i) The shape of the gust is—

$$U = \frac{U_{de}}{2}\left(1\text{-COS}\frac{2\pi s}{25C}\right)$$

Where—

s = Distance penetrated into gust (ft.);

C = Mean geometric chord of wing (ft.); and

Ude = Derived gust velocity referred to in sub-paragraph (1) of this section.

The gust shaping in §23.333(c)(2)(i) addresses the idea that real world gusts are not true step functions but take distance to shear from the calm air and the response of the aircraft depends on how far into the gust the aircraft has progressed. Reference 2 states "Some specifications require gust velocities of 50 ft/sec, with corresponding gust reduction factors *K* of about 0.6. Since the values of *KU* in this case are also about 30 ft/sec, the net effect is equivalent to a gust velocity *U* of 30 ft/sec with a *K* of 1.0." Thus, our choice to calculate the gust response with a 30 ft/sec sharp edged gust is consistent with the guidance in 14 CFR Part 23.

Thus, V_B is the airspeed at which a 30 ft/sec sharp edged gust will create sufficient incremental load to increase the load factor to the limit load factor.

A Bearhawk Example

At this point, I had hoped to throw up a V-n diagram for the Bearhawk and show how the gust analysis fit on the diagram. Of course, nothing ever seems to be that simple. The first issue comes from the fact that no official V-n diagram has been published for the Bearhawk.

In the Spring 1995 edition of Bear-Tracks, the newsletter for builders of the Barrows Bearhawk, it was stated that the design maneuvering speed was 110 mph calibrated airspeed at 5 g load factor. Within an inch of this statement was another statement that the maximum load factor (limit load factor) was 4.5 g at a gross weight of 2400 pounds, and 5 g at a gross weight of 2300 pounds. These limit load factors put the airplane in the utility category.

Since 5 g was the limit load factor at 2300 pounds, it was assumed that the design maneuvering speed of 110 mph (95.6 KCAS) was for a gross weight of 2300 pounds. For a stall at 5 g, 110 mph and 2300 pounds, the $C_{L_{max}}$ would be 2.06. However, flight tests of Bearhawk #164 in 2010 measured a Flaps Up $C_{L_{max}}$ of 1.46. I started searching for a reasonable way to resolve these differences.

A review of the old 14 CFR 23.337 found the following recommended load factors.

Category	Positive Limit Load Factor	Negative Limit Load Factor
Normal	3.8	-1.52
Utility	4.4	-1.76

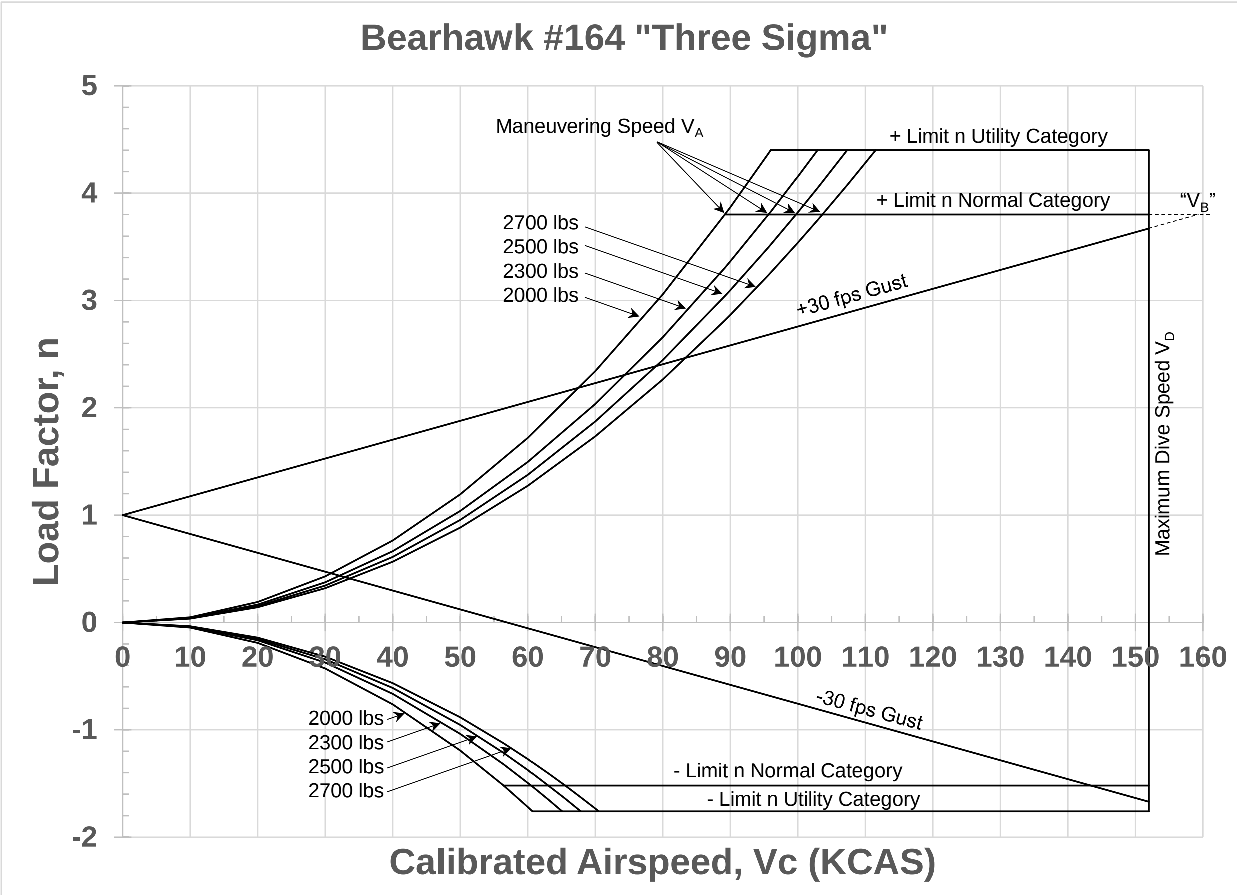
These are minimum requirements, and the designer can always choose to exceed these recommendations.

After some recreational maths, I determined that a Flaps Up stall at 3.8 g (normal category), 110 mph (95.6 KCAS) and 2300 pounds would give a $C_{L_{max}}$ of 1.56. A $C_{L_{max}}$ of 1.56 is close to the measured $C_{L_{max}}$ of 1.46. The difference in 1 g stall speed would be less than 2 knots, which was within a reasonable uncertainty of airspeed measurement.

Therefore, I assumed that the positive $C_{L_{max}}$ was 1.56 for calculating the lift limit lines for the V-n diagram. I also assumed that the negative $C_{L_{max}}$ was -1.56 in the absence of any other data. These were used to draw lift limit lines for gross weights of 2000, 2300, 2500, and 2700 pounds.

The maximum dive speed (V_D or V_{ne}) is published at 175 mph calibrated airspeed (152 KCAS).

The resulting Bearhawk V-n diagram is presented here. This may not match what the designer considers the V-n diagram to look like, but this V-n diagram matches the performance of Bearhawk #164.



This figure shows a design maneuver point at 95.6 KCAS for 2300 pounds and 3.8 g. It also shows the counterintuitive result that the maneuver point airspeed decreases as gross weight decreases, as covered in Reference 5.

This figure also shows the gust response for a 30 ft/sec sharp edged gust, positive and negative. The design speed for maximum gust intensity (V_B) is defined by where the gust response line intersects the limit load factor. In this case, the intersection is at 159 KCAS. However, this is in excess of the maximum dive speed (V_D). Therefore, for this airplane V_B is undefined.

I'm Tired of Getting Kicked in the Butt

That's a lot of fun and interesting maths, but how should we use this new knowledge to make piloting decisions? At the risk of being declared a wimp in the pilot community, I really don't like being in turbulence, especially in an airplane that feels less stable than a typical Cessna. The analysis above tells me that my Bearhawk should be safe for any reasonable gust seen in operation. Still, we know that the incremental load factor caused by a gust changes linearly with airspeed. Thus, slowing down to maneuvering speed reduces the perceived bumps while giving an excess of safety from exceeding the limit load factor.

If the size of the turbulence bumps becomes uncomfortable, you can increase your comfort and your passengers' comfort by slowing down. Maintain a sufficient cushion above stall speed. Slowing to design maneuvering speed is a good compromise.

- Russ Erb

References

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